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# Vortex free energy and deconfinement in center-blind discretizations of Yang-Mills theories

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## Abstract

Maximal 't Hooft loops are studied in  $SO(3)$  lattice gauge theory at finite temperature  $T$ . Tunneling barriers among twist sectors causing loss of ergodicity for local update algorithms are overcome through parallel tempering, enabling us to measure the vortex free energy  $F$  and to identify a deconfinement transition at some  $\beta_A^{crit}$ . The behavior of  $F$  below  $\beta_A^{crit}$  shows however striking differences with what is expected from discretizations in the fundamental representation.

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Topology plays an important rôle in the non-perturbative dynamics of Yang-Mills theories. In particular the vacuum condensation of topological excitations might explain quark confinement and the existence of a mass gap through various scenarios described in the literature [1, 2, 3]. Some of these models allow to define a topological order parameter for the finite  $T$  deconfinement transition; for 't Hooft magnetic vortices, classified by the first homotopy class  $\mathbb{Z}_N$  of the continuum Yang-Mills gauge group  $SU(N)/\mathbb{Z}_N$  [3], the change in free energy  $F = \Delta U - T\Delta S$  for their creation might play such rôle and has received broad attention, in particular in lattice discretizations at zero and finite  $T$  [4, 5, 6]. The main problem in such non-perturbative regularizations is that creating a vortex is equivalent to the introduction of a non-trivial twist. For discretizations in the fundamental representation, transforming under the *enlarged* gauge group  $SU(N)$  ( $N = 2$  in this paper), this cannot be implemented dynamically but only via a modification of their boundary conditions (b.c.), the generalized partition function  $\tilde{Z}$  being defined through the weighted sum of partition functions with fixed twisted b.c. Since each of them must be determined by independent simulations their relative weights can only be calculated through indirect means [5, 6].

Universality arguments are often cited to claim that results in lattice Yang-Mills theories will not depend on the discretization chosen. A natural alternative in calculating  $F$  would therefore be to directly discretize the theory in the  $SU(N)/\mathbb{Z}_N$  representation ( $SO(3)$  in our case) with periodic b.c., naturally transforming under the continuum Yang-Mills gauge group [7]. The adjoint partition function  $Z(\beta_A)$  should in fact be equivalent to  $\tilde{Z}$  provided the  $SO(3)$  native constraint  $\sigma_c = \prod_{\bar{P} \in \partial c} \text{sign}(\text{Tr}_F U_{\bar{P}}) = 1$  is satisfied for every elementary 3-cube  $c$ , where  $U_{\bar{P}}$  denotes the plaquettes belonging to its surface  $\partial c$  [7, 8]:  $\mathbb{Z}_2$  magnetic monopoles are suppressed and only closed  $\mathbb{Z}_2$  magnetic vortices winding around the boundaries are allowed, i.e. in this limit adjoint actions dynamically allow all topological sectors which in the fundamental case must be fixed through twisted b.c.. Moreover, since the standard spontaneous center symmetry breaking argument for the deconfinement transition does not apply to the center-blind adjoint discretization the question whether  $F$  behaves as an order parameter is of major interest, also in light of recent studies for alternative descriptions of confinement in centerless theories [9].

A practical obstacle one needs to overcome in investigating the adjoint theory is the appearance of a bulk transition at some  $\bar{\beta}_A$ , separating a strong coupling chaotic phase (I) continuously connected with the fundamental action, where  $\langle \sigma_c \rangle \simeq 0$ , from a weak coupling ordered phase (II) where  $\langle \sigma_c \rangle \simeq 1$  [10, 11, 12]. In phase II, where one wishes to exploit the relation between  $Z(\beta_A)$  and  $\tilde{Z}$  mentioned above, high potential barriers separating twist sectors suppress tunneling among them for local update algorithms [7]. On the other hand a well-known result is that a center blind  $\mathbb{Z}_2$  monopole suppression term in the action  $\lambda \sum_c (1 - \sigma_c)$  weakens the order of the bulk transition while moving it down into the strong coupling region (see the curved dashed line in Fig. 1a) [12]. For asymmetric lattice sizes  $N_\tau \times N_s^3$ ,  $N_\tau \ll N_s$  indications for a finite temperature critical line  $\beta_A^{crit}(\lambda, N_\tau)$  within phase II (horizontal dashed line in Fig. 1a for  $N_\tau = 4$ ) have already been found from simulations at *fixed* twist [13, 14, 15, 16].

In this paper we solve the problem of ergodic updates through parallel tempering (PT) [17, 18] for the adjoint Wilson action with  $\mathbb{Z}_2$  monopole suppression

$$S = \beta_A \sum_P \left( 1 - \frac{1}{3} \text{Tr}_A U_P \right) + \lambda \sum_c (1 - \sigma_c), \quad (1)$$

where  $U_P$  denotes the standard plaquette variable and  $\text{Tr}_A O = (\text{Tr}_F O)^2 - 1 = \text{Tr}_F(O^2) + 1$

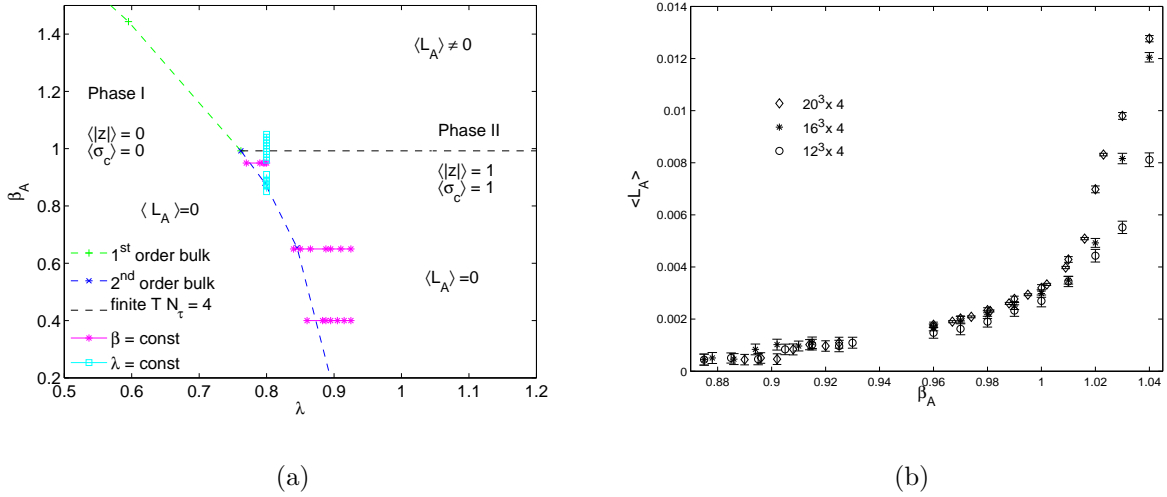


FIG. 1: (a) Phase structure in the  $\beta_A$ - $\lambda$  plane with paths of couplings indicated as used in our simulations. (b) Adjoint Polyakov loop average  $\langle L_A \rangle$  versus  $\beta_A$  at fixed  $\lambda = 0.8$  and  $N_\tau = 4$ .

the adjoint trace. Our PT paths in the  $\beta_A - \lambda$  plane extend over the *second order* leg of the bulk transition straight into the inner region of phase II as shown in Fig. 1a, with two paths at fixed  $\beta_A$ , one path with fixed  $\lambda$ , and one path consisting of a fixed- $\beta_A$  and a fixed- $\lambda$  part. In this way we avoid the necessity to cross high potential barriers. This enables us to account for all twist sectors, restoring ergodicity, and to study  $F$  at finite  $T$ . As a qualitative indicator of deconfinement we have also measured the adjoint Polyakov loop  $\langle L_A \rangle = \langle \sum_{\vec{x}} \text{Tr}_A L(\vec{x}) \rangle / (3N_s^3)$ ,  $L(\vec{x}) = \prod_t U_4(\vec{x}, t)$ , which strictly speaking cannot behave as an order parameter for the deconfinement transition, delivering at most information on the screening length for the effective adjoint potential (see Fig. 1b). A preliminary report on the present investigation was presented in [19]; further results on other observables as well as a detailed description of the implementation of the algorithm will appear in a separate publication [20]. To our knowledge this is the first successful attempt to study the deconfinement transition of the  $SO(3)$  lattice gauge theory via an ergodic simulation at large volume as well as the first determination of  $F$  in the confined phase of a center-blind discretization of Yang-Mills theories.

In PT [17, 18] we update  $K$  configurations  $\mathcal{F}_i$ ,  $i = 1, \dots, K$ , with couplings  $(\lambda, \beta_A)_i$  swapping neighboring pairs from smaller to larger values according to a Metropolis acceptance probability, satisfying detailed balance,  $P_{\text{swap}}(i, j) = \min[1, \exp(-\Delta S_{ij})]$ , where

$$\Delta S_{ij} = S[(\lambda, \beta_A)_i, \mathcal{F}_i] + S[(\lambda, \beta_A)_j, \mathcal{F}_j] - S[(\lambda, \beta_A)_i, \mathcal{F}_j] - S[(\lambda, \beta_A)_j, \mathcal{F}_i]. \quad (2)$$

Compared to other methods [21, 22] the appeal of PT is its easy implementation both at criticality and away from it, needing no knowledge of re-weighting factors or other dynamical input. This is a welcome property for us since we wish to go as far as possible from the bulk transition into phase II. In view of various experiences with simulated tempering one also expects PT to be more efficient than multi-canonical simulations. For the success of the method in the case under consideration the softening of the bulk transition to 2<sup>nd</sup> order is crucial, since we “transport” tunneling from phase I at lower  $\lambda$  into phase II at larger

$\lambda$ , where twist sectors are well defined but frozen. To work at low  $\lambda$ , i.e. through a 1<sup>st</sup> order bulk transition, would make barriers too high and kill any hope of ergodicity at large volume, as experienced in [7] for  $N_s > 8$ .

Some care is of course necessary also with our method. In particular to maintain a sufficient swapping acceptance rate  $\omega$  the distance between neighboring couplings must diminish with the volume. On the other hand to keep cross-correlations under control one does not wish the acceptance rate to be too high. We have chosen to tune the parameters for each path and volume at hand so to keep the acceptance rate roughly fixed at around  $\omega = 12\%$ , a value for which we empirically find a good balance between auto- (in the sense of freezing of the sectors) and cross-correlations. For the paths in Fig 1a contributing to the left and right branches in Fig 2a details on the statistics, i.e. number of configurations  $N$  and number of ensembles  $K$ , are given in Table I. The paths at fixed  $\beta_A = 0.4$  and  $0.65$  for  $N_s = 16$  shown in Fig 2b were calculated with  $K = 7$  and  $N = 30000$  configurations. To remain on the safe side, in Fig 2a we have chosen not to quote the ensembles related to the end points of the paths since, having no further configurations to swap with, they might be affected by systematic errors. We wish however to stress that this has never been observed in the PT literature and we also have no indication that this might be the case. Further details on the algorithm and a detailed analysis of correlations will be reported in a forthcoming paper [20]. For the fixed  $\lambda$  paths of Fig. 1a, along which main simulations have been performed,

$N_s$	$K$	$N$
12	10	30000
16	10	30000
20	10	30000
24	10	30000

(a)

$N_s$	$K$	$N$
12	14	100000
16	14	100000
20	14	100000
24	10	100000

(b)

TABLE I: Statistics achieved for the data given in Fig 2a. The left branches correspond to (a) while the right branches together with pieces at fixed  $\beta_A = 0.95$  and varying  $\lambda$  correspond to (b).

we can fit very well the step  $\delta\beta_A$  needed to keep  $\omega$  fixed with a law of the form

$$\delta\beta_A(\omega, N_s) \simeq \frac{\alpha(\omega)}{N_s^2}, \quad (3)$$

where  $\alpha(12\%) = 2.15(3)$  in the  $\beta_A = 0.95 - 1.09$  range considered, although we expect it to change with the  $(\lambda, \beta_A)$  window. Such scaling implies that in order to explore a fixed region  $\Delta\beta_A$  of parameter space the number of ensembles will scale like

$$K \simeq \frac{\Delta\beta_A}{\alpha} N_s^2. \quad (4)$$

Therefore we cannot go too deep into phase II since the number of PT simulations will eventually become too large for the computational means at our disposal.

The native  $SO(3)$  temporal twists are given by  $z_i \equiv N_s^{-2} \sum_{x_j, x_k} \prod_{P \in \text{plane } i,4} \text{signTr}_F U_P$ , ( $\epsilon_{ijk4} = 1$ ) for  $i = x, y, z$  [7, 14]. Within phase II they are well defined having values close

to either 1 or -1 for each configuration. The partition functions restricted to fixed twist are given as expectation values  $\langle \nu_k \rangle = Z|_{z=k}/Z$  of the projectors [3]

$$\begin{aligned} \nu_0 &= \frac{1}{8} \prod_{i=x,y,z} [1 + \text{sign}(z_i)], & \nu_1 &= \frac{1}{8} \sum_{j=x,y,z} \prod_{i=x,y,z} [1 + (1 - 2\delta_{i,j}) \text{sign}(z_i)], \\ \nu_2 &= \frac{1}{8} \sum_{j=x,y,z} \prod_{i=x,y,z} [1 - (1 - 2\delta_{i,j}) \text{sign}(z_i)], & \nu_3 &= \frac{1}{8} \prod_{i=x,y,z} [1 - \text{sign}(z_i)]. \end{aligned} \quad (5)$$

The 2- and 3-twist sectors can of course only exist on  $\mathbb{T}^3$  [3, 6]. From Eq. (5), since for an adjoint theory a change of twist sector leaves the action unchanged,  $\Delta U = 0$  and:

$$F = -T \log \frac{Z_1}{3Z_0} = -\frac{1}{aN_\tau} \log \frac{\langle \nu_1 \rangle}{3\langle \nu_0 \rangle}. \quad (6)$$

The factor three in the denominator is again due to the three equivalent 1-twist sectors on  $\mathbb{T}^3$  rather than one as on  $\mathbb{R}^3$  [6, 7]. With such a choice  $F$  will be zero if all twists are equally probable, i.e. on top of the bulk transition and everywhere in phase I, where twist sectors are however ill-defined due to the presence of open vortices. Eq. (6) obviously implies  $F = 0$  in the  $T \rightarrow 0$  limit [3, 4, 5] as long as the  $\Delta S$  contribution remains bounded. Fig. 2a shows numerical results for  $F/T$  in lattice units at fixed  $\lambda = 0.8$  obtained along the two separated paths indicated in Fig. 1a. Errors are given combining statistical errors with auto and cross correlations. The data start on top of the 2<sup>nd</sup> order bulk transition and go upward to what

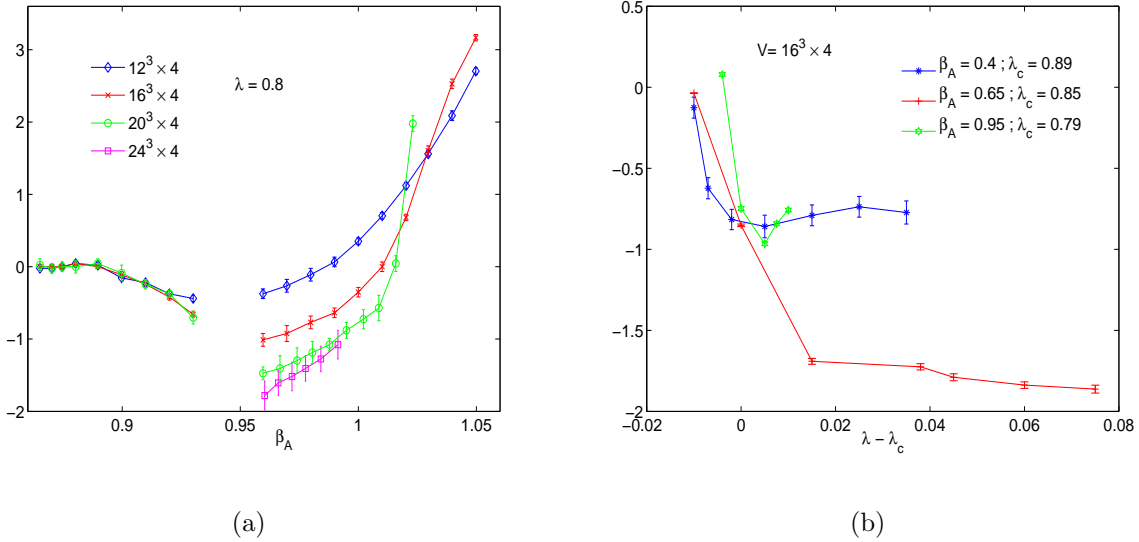


FIG. 2: (a)  $aN_\tau F$  versus  $\beta_A$  at  $\lambda = 0.8$  and various  $N_s$ . (b)  $aN_\tau F$  versus  $\lambda - \lambda_c$  at fixed  $\beta_A$  for  $N_s = 16$ .  $\lambda_c(\beta_A)$  denotes the position of the bulk transition.

we interpret as the finite  $T$  deconfinement transition – given the rapid growth of  $\langle L_A \rangle$  (cf. Fig. 1b) as well as of  $F$  and taking fixed-twist results for varying  $N_\tau$  into account [14, 15, 16].

The behavior in phase I and just at the bulk transition as well as in the deconfined phase at large  $\beta_A$  (upper phase II) is in agreement with the standard vortex arguments

for confinement [3]: if vortices behave "chaotically" then  $F$  should be zero and the theory confines, while as deconfinement occurs  $F$  should rise as  $F \sim \tilde{\sigma} N_s^2$ ,  $\tilde{\sigma}(T)$  being the dual string tension. This is qualitatively in agreement with our data. While  $F \approx 0$  close to the bulk transition, we find a strong rise of  $F$  for  $\beta_A \geq \beta_A^{crit} = 1.01(1)$  allowing to locate the finite-temperature transition in phase II. As already explained above, we are not able to go too deep into the deconfined phase with the computing facilities at our disposal. Thus, we cannot really check whether the data for  $\beta_A \gg \beta_A^{crit}$  are consistent with the expected  $O(N_s^2)$  plateaus or whether they get saturated with respect to the thermodynamic limit, i.e. whether  $\tilde{\sigma}$  can indeed be calculated. The effort necessary to this purpose, assuming that the estimate in Eq. (4) still works at higher  $\beta_A$  and even taking into account that for higher volumes the asymptotic behavior should kick in earlier, we would need to simulate around 50 parallel ensembles for each volume, again for a statistics of at least  $O(10^5)$  per configuration in each ensemble. For volumes with  $N_s \geq 20$ , for which finite size effects start to be reasonably small, this goes beyond the computational power at our disposal, although it should be manageable with a medium sized PC cluster.

For  $\beta_A \leq \beta_A^{crit}$ , i.e. throughout the confining region of phase II, we find negative values for  $F$  which stabilize at large 3-volumes ( $N_s \geq 20$ ).  $F < 0$  comes as a surprise, meaning that vortex production is enhanced as compared to phase I. This is in contrast to what expected from arguments valid within the fundamental representation, i.e.  $F(T) = 0$  throughout the confined phase. For an independent check we have carried out simulations at a few (fixed) lower  $\beta_A$ -values, varying  $\lambda$  i.e. at somewhat lower temperatures (see the horizontal paths drawn in Fig. 1a). The results are plotted in Fig. 2b. Again we find  $F < 0$ , but the plateau values do not behave monotonously as a function of  $\beta_A$ . Passing some minimum they increase again for decreasing  $\beta_A$ . This is compatible with the expectation that the free energy should go to zero in the zero-temperature limit [3, 4, 5]. A systematic extrapolation for different volumes and  $N_\tau$  would be required to confirm this behavior and to decide whether  $F/T$  itself vanishes or goes to a constant value.

Let us draw the conclusions. The main result of the present paper is the success in sampling the *full* partition function via ergodic PT Monte Carlo simulations and determining the free energy  $F$  for the creation of a  $\mathbb{Z}_2$  vortex in pure  $SO(3)$  Yang-Mills theory at finite  $T$ . We have seen a clear indication for a deconfinement transition consistent with earlier findings of a second order transition at fixed twist. Furthermore we find that  $F$  does not vanish in the confined phase at  $T \neq 0$ , vortex creation being enhanced throughout it;  $F$  cannot therefore serve as an order parameter in a strict sense. This implies that the adjoint theory is unable to exhibit an order parameter for center symmetry breaking in any form, much like in the case of strictly centerless groups [9]. This is not in contradiction with the confining properties of the model,  $F = 0$  being a sufficient but not necessary condition for confinement away from the  $T = 0$  limit [3, 23]. Moreover while vortex suppression for  $T < T_c$  would have been difficult to justify in light of the literature [24] the vortex enhancement we observe does not contradict that they can play a rôle in describing confinement, although one cannot speak of vortex condensation in the usual understanding.

Let us finish with a short remark on the universality problem we seem to face. When identifying the partition functions  $\tilde{Z} \simeq Z(\beta_A)$ , invoking universality for observables, one should be cautious. First of all, although  $\mathbb{Z}_2$  monopoles become suppressed in the continuum limit of  $\tilde{Z}$ ,  $\langle \sigma_c \rangle$  is still far from unity for the range of parameters commonly used in the simulations [4, 5, 6] and open vortices might still dominate the partition function. Moreover  $Z(\beta_A)$  simply does not allow to define physical observables in the fundamental

representation. Expectation values of fundamental Wilson and Polyakov loops and all their correlators vanish identically for all  $\beta_A$ , i.e. a fundamental string tension cannot be defined in a straightforward way. To our knowledge bounds for  $F$  from reflection positivity or the connection with the electric flux have only been derived within the fundamental representation [3, 6, 23]. An interpretation of universality implying that any observable will assume the same value in any discretization is therefore trivially contradicted by the above considerations. However, a slightly more conservative reading can agree with our findings: the truly physical properties measurable in “experiments” like glueball masses and the critical exponents at the transition should be reflected by physical observables which can be defined *irrespective* of the discretization chosen.

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